

## 7 Mex

The earlier work showed that the key to playing these “take away” games correctly is having a Nimber for every position. I now want to describe a method for assigning Nimbers to game positions.

The key mathematical tool is called the Mex. This is another funny word (like Nimber). Mex stands for **Minimum Excluded Value**. (Memorize this NOW; we’ll use it a lot.) Here’s what we mean. Suppose in some game position, whose value you do not know, you can move to positions whose Nimber values are 0, 1, 3, and 5. I.e. The choices are in the set  $\{0,1,3,5\}$  The smallest non-negative integer not in that set is 2. Thus we say the Mex of that set (or the Mex for that position) is 2. Thus 2 will be the value of that position.

Let’s look at the game of Take-2-3-5. This is the same as Take-1-2-3 except here we can take 2 or 3 or 5.

Let’s try to fill in this table using the Mex principle:

Size of pile	0	1	2	3	4	5	6	7	8
Nimber value									

The first two values are 0 because the player on move in piles of 0 or 1 has no moves.

Size of pile	0	1	2	3	4	5	6	7	8
Nimber value	0	0							

The value of pile size 2 then follows because a move to 0 size is the only move. Hence the Mex is 1.

Size of pile	0	1	2	3	4	5	6	7	8
Nimber value	0	0	1	1					

We can now fill in size 3. There are two legal moves both to positions of Nimber value 0. So the Mex is 1 .

Try to find the Nimber values of the rest of this table

Size of pile	0	1	2	3	4	5	6	7	8	9	10	11
Nimber value	0	0	1	1								