Here is a fairly complete analysis of the game "I've Got A Picture"

Rules for "I've Got A Picture"

- Player A draws a card from a shuffled deck.
- Then A bets either \$1 or \$5. Player B can now concede the bet or double the bet.
- If the bet is doubled, the card is then exposed and player A wins twice the amount bet if the card is a Jack, Queen or King. If the card is A, 2, 3, 4, 5, 6, 7, 8, 9, 10, then B wins twice the amount bet .
- If the bet is conceded then player A wins the amount bet.

Key Principle for Games Which Involve Bluffing:

Don't Get Outguessed

Some observations for "I've Got A Picture"

Before starting let's agree to refer to player A (who has the cards and makes the bet) as Alice and player B as Bill and let's use "+" to mean Alice gets paid (by Bill) and "–" to mean Bill get's paid (by Alice).

1. First assumptions
   - Alice will always bet \$5 with a picture. Why?
   - Bill will double every bet of \$1. Why?
2. If Alice bets \$5 when she has a picture and \$1 when she has a non-picture, she will lose in the long term if Bill doesn't double any bets of \$5. Of course, Bill does double bets of \$1.

   $$EV=(3/13)(+5) + (10/13)(-2)= -5/13$$

3. If Alice bets \$5 on non-pictures too often she may also lose. For example suppose she bets \$5 whenever she has an A or a 2 and on all J, Q, K, and Bill always doubles bets of \$5. Then

   $$EV=(3/13)(+10)+(2/13)(-10)+(8/13)(-2)= -6/13$$

   (Notice that Bill has used two different strategies to defeat Alice in 2. and 3. How does Bill know what to do? We'll answer that question later.)
4. *But if Alice bets \$5 on 1/10 of the non-picture cards she is guaranteed to be a long term winner.*

   To see this we need one more new idea:

   I'll call any strategy where a player always does the same thing a "pure strategy". In this game a pure strategy for Bill might be: Always double bets of \$5 and \$1. His other pure strategy is: Never double bets of \$5

but still double bets of $1. These are both "pure" because no element of randomness is involved. Let's see how Bill fares against Alice's 1/10 bluffing strategy. (Since there are 10 non-pictures, Alice can implement her strategy by bluffing with an Ace (or any other non-picture) but this is not essential, she could also use any system which randomly says "bluff" 1/10 of the time she gets a non-picture.)
Against Bill's "Always double" strategy we get:

EV=(3/13)(+10) + (1/10)(10/13)(-10)+(9/13)(–2)= **2**/13

Against Bill's "Never double" strategy we get:

EV=(3/13)(+5) + (1/10)(10/13)(+5) +(9/13)(-2)= **2**/13

Since Bill's two pure strategies yield the same result, any combination of these will yield the same result.

So Alice has turned a game that looked bad for her (see 1,2 and 3) into a game which is profitable for her by bluffing a fraction of the time!

We've used the "Don't Get Outguessed" principle: Find a strategy which gives the same results against your opponent's pure strategies and then you won't be outguessed.

Analysis Using DGO

Now we solve two problems. How did Alice know that 1/10 was the correct bluffing percentage? What should Bill do so *he* won't be outguessed?

5. Using these assumptions, we let p be the probability that Alice bluffs with a loser. Then we calculate the expected value to Alice against two types of player: the "always double bets of $5 player" and the "never double bets of $5 player". If we use the value for p obtained when these are equal, we have found a strategy for A that prevents her from being outguessed.
    - EV against "always double $5" =
      (3/13)(+10)+p(10/13)*(–10)+(1–p)(10/13)(–2)
    - EV against "never double $5" =
      (3/13)(+5)+p(10/13)(+5)+(1–p)(10/13)(–2)
    - Then setting the two expressions equal we get p=1/10 and that the expected value for Alice is 2/13.
    - Note: Since we've found a value for p that gives the same expected value for Alice against "always double" and "never double" Alice's expected value will also be the same against any combination of doubling and not doubling bets of $5.

6. OTOH we can also calculate a strategy for Bill. Let s be the probability that Bill doubles bets of $5. We now calculate the expected value to Bill against two strategies: “always bet $5 with a loser” and “never bet $5 with a loser”. With EVB analogous to EVA we get:
   - EV against “always bluff” =

     (3/13)s(+10) + (3/13)(1–s)(+5) +
     (10/13)s(–10)+(10/13)(1–s)(+5)

     (Note these four terms represent, in order: A gets picture and B doubles, A gets picture and B concedes, A gets loser and A bluffs and B doubles, and lastly A gets loser and A bluffs and B concedes.)

   - EV against “never bluff” =

     (3/13)(+10)s+(3/13)(1–s)(+5)+(10/13)(–2)
   - Then setting the two expressions equal we get s=7/15 and that the expected value for Bill is –2/13. (I.e. B loses 2/13)
   - Thus the strategy for Alice guarantees a win of at least 2/13 and the strategy for Bill a loss of at most 2/13.
   - Note: Since we’ve found a value for s that gives the same expected value for Bill against “always bluff” and “never bluff” B’s expected value will also be the same against any combination of Alice bluffing or not bluffing with losers.
7. Notice that if either player deviates from these strategies the opponent can take advantage. We saw in 2. and 3. that Alice can lose by bluffing too often or too seldom. Similarly if Bill doubles too often or too seldom, Bill will lose more than –2/13 per game.
8. Since this game favors Alice by 2/13 per game, she should be willing to pay up to 2/13 for the privilege of playing.