Take and Break Games

“Take and Break” games are something like Nim. However instead of viewing a Nim position as consisting of piles of matches, we view the position as consisting of rows of matches which can be broken into smaller rows. Or we view a pile as capable of being divided into two smaller piles.

Here’s an example of such a game.

1. We start with a strip of candy, one unit wide, which is divided into $n$ 1x1 squares.
2. The main rule is that the players can eat (i.e. take) a square only when the square eaten is NOT an end of a strip. At their turn a player can only eat exactly one square.
3. Each time a square is eaten the strip from which it was taken is divided into two smaller strips.
   For example, if the first player in this game ate the 4th square, we’d now have a game in which there are two strips, one with three squares and the second with six squares.

   Here’s a picture of a starting position with 10 squares and the new position after one move.

   

4. The game ends when all the strips that remain are either single squares or pairs of squares. This is because then there are no legal moves.
5. The winner is whoever ate the last square.
Take and Break – Details

Here are the details for how to analyse a Take and Break Game

1. Start with the pile of size 0. That is clearly a “zero position” and has Nimber value \(0^*\).

2. Then take the larger piles in increasing order. You will apply “Mex” to each pile.

3. If at any time while we trying to apply Mex we need to evaluate a move which breaks the pile into two smaller piles then the evaluation is done as follows:
   a. Find the nimber value of each of the smaller piles
   b. Add them together to find the value of the move that broke them

Here’s an example.
Suppose we’re analyzing a game and we’ve got the following table:

<table>
<thead>
<tr>
<th>Pile size</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nimber value</td>
<td>(0^*)</td>
<td>(1^*)</td>
<td>(1^*)</td>
<td>(2^*)</td>
<td>(3^*)</td>
<td>(2^*)</td>
<td></td>
</tr>
</tbody>
</table>

which we want to extend to include a pile of 6.
Now suppose the legal moves from a pile of 6 are:
5, 4, 4+2, 3+3 . (Don’t worry about what game this actually is.) That is suppose we can take 1 or 2 or break into 4+2 or into 3+3.
The Mex will be the smallest value not in:
\(\{2^*, 3^*, 3^*+1^*, 2^*+2^*\} = \{2^*, 3^*, 2^*, 0^*\} = \{2^*, 3^*, 0^*\}\)
Thus 6 is evaluated as \(1^*\).