Math 190

Counting Poker Hands

Poker: Poker is played with a 52 card deck. Each card has two attributes, a rank and a suit. The rank of a card can be any of 13 possibilities:

$$\{2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, K, A\},$$

while the suit can be any of 4 possibilities:

$$\{\clubsuit, \diamondsuit, \heartsuit, \spadesuit\}.$$

We exclude jokers and wildcards.

Purpose: The purpose of this handout is to review the counting techniques we applied in class to count the number of each of the various types of poker hands.

Example 1. The number of different 5 card poker hands was our first example. This is a standard combinations question and the solution is $\binom{52}{5} = 2,598,960$.

Straight flush: A straight flush is a hand with 5 consecutive ranks, all of which are of the same suit. Straights may begin with an ace, two, ... , 10. Thus there are $\binom{10}{1}$ ways to begin the straight and $\binom{4}{1}$ ways to select the suit, for a total of $\binom{10}{1}\binom{4}{1} = 40$ possible straight flushes. Note that 4 of these are the royal flushes (10, J, Q, K, A), which we counted separately.

Four of a kind: This is 4 cards of the same rank. The 5th card is necessarily of a different rank. Our computation is: $\binom{13}{1}$ ways to select the rank, and $\binom{4}{4}$ ways to select the 4 cards of that rank. The last card can be selected in $\binom{48}{1}$ ways (any card of another rank). Thus, by the multiplication principle:

$$\binom{13}{1}\binom{4}{4}\binom{48}{1} = 624.$$

Full house: This consists of three cards of one rank and two cards of another rank. Thus, the number of such hands is:

$$\binom{13}{1}\binom{4}{3}\binom{12}{1}\binom{4}{2} = 3744,$$

Flush: This consists of 5 cards in one suit (but not a straight). Thus, we choose the suit, then the 5 cards, then subtract the 40 straight flush hands computed above:

$$\binom{4}{1}\binom{13}{5} - 40 = 5108.$$

Straight: This consists of 5 consecutive ranks (not all of one suit). As noted above, there are $\binom{10}{1}$ ways to select the straight (equivalent to selecting the starting card). Then, there are $\binom{4}{1}$ ways to select the card in each of the 5 positions of the straight. But this over counts as it allows a straight flush, so we need to subtract that value. Our computation is then:

$$\binom{10}{1}\binom{4}{1}^5 - 40 = 10,200.$$

Three of a kind: Consists of three cards of one rank and two more cards, of two further ranks. Thus, our computation is:

$$\binom{13}{1}\binom{4}{3}\binom{12}{2}\binom{4}{1}^2 = 54,912.$$

Two Pair: Consists of one pair of one rank and a second pair of another rank and a 5th card of yet a third rank. Thus, by the multiplication principle our count becomes:

$$\binom{13}{2}\binom{4}{2}^2\binom{44}{1} = 123,552.$$

One pair: Consists of 2 cards of one rank and three other cards that are of three other ranks. By the mullt. principle the count is:

$$\binom{13}{1}\binom{4}{2}\binom{12}{3}\binom{4}{1}^3 = 1,098,240.$$

High card: Consists of all other hands. So by subtracting we have:

$$2,598,960 - 40 - 624 - 3744 - 5108 - 10,200 - 54,912 - 123,552 - 1,098,240 = 1,302,540.$$

HAND	HOW TO COUNT	NUMBER
Straight Flush	$\binom{10}{1}\binom{4}{1}$	40
Four of a kind	$\binom{13}{1}\binom{48}{1}$	624
Full house	$\binom{13}{1}\binom{4}{3}\binom{12}{1}\binom{4}{2}$	3744
Flush	$\binom{13}{5}\binom{4}{1} - 40$	5108
Straight	$\binom{10}{1}\binom{4}{1}^5 - 40$	10,200
Three of a kind	$\binom{13}{1}\binom{4}{3}\binom{12}{2}\binom{4}{1}^2$	54,912
Two Pair	$\binom{13}{2}\binom{4}{2}^2\binom{48}{1}$	123,552
One pair	$\binom{13}{1}\binom{4}{2}\binom{12}{3}\binom{4}{1}^3$	1,098,240
High card	$\binom{52}{5}-$ all other counts	1,302,540

3 Card Poker

This is poker played with only 3 cards. Using the rules you have learned and the experience from computing the number of 5 card poker hands to complete the table below. Try to justify your computations as we did before.

HAND	HOW TO COUNT THE NO. of HANDS	NUMBER
Straight flush		
Three of a kind		
Straight		
Flush		
One pair		