

HW2 – Fall 2013 – Abstract Algebra

You may work with at most two other people. Each person MUST write up their own answers without their teammates being present. The answers won't be exactly the same because they weren't copied. Be sure your name appears and the names of any teammates are also made clear. If you choose to work alone, I will give that extra credit! Groups should not consult and if you get help from any source (say book, web, faculty, etc.) that must be cited.

Recall that given two (or more) groups that we can define a new group by taking letting the underlying set be the Cartesian product of the two groups and defining the group operation on the product "coordinatewise".

1. Prove this is a group. A great proof would do this for sets of groups of any size but if you do it for just two groups that's OK too.
2. Let G_1 be $\{1, -1\}$ under ordinary multiplication.
Let both G_2 and G_3 be $\{0,1,2\}$ under multiplication mod 3.

Let $G = G_1 \times G_2 \times G_3$

- a. Find $|G|$
- b. How many elements, if any, have order 2? List them.
- c. How many elements, if any, have order 3? List them.
- d. How many elements, if any, have order 6? List them.
- e. Are there any elements of orders other than 2, 3, and 6? If so, how many? List them.
- f. Is G a cyclic group? Why or why not?
- g. List one subgroup of as many different orders as possible. I.e. One subgroup of order 1, one of order 2, etc.

Hint: The answers to b, c, d, and e should total the answer to a.