

Suppose that Z and W are two non-zero complex numbers. Can $W \cdot Z$ be zero?

Let $Z = a + bi$ and $W = c + di$ Then $ZW = ac - bd + (ad + bc)i$.

Define: $|Z| = \sqrt{a^2 + b^2}$ and note $|ZW| = |Z| |W|$

which we prove as follows.

We use squares which saves us constantly writing radical signs. Then we just take square roots at the end.

$$\begin{aligned} |ZW|^2 &= \\ (ac - bd)^2 + (ad + bc)^2 &= \\ a^2c^2 - 2acbd + b^2d^2 + a^2d^2 + 2adbc + b^2c^2 &= \\ a^2c^2 + b^2d^2 + a^2d^2 + b^2c^2 &= \\ (a^2 + b^2)(c^2 + d^2) &= \\ |Z|^2 |W|^2 \end{aligned}$$

Since the right hand side is positive, so is the left hand side.

The other item was to show that for integers a , b and c that if ab is a square and bc is a square then ac is a square. Proving this involves more work than I realized and I shouldn't have asked this question.