Conjugacy in S_n

We say two elements, say a and b, in a group G are *conjugate* if there exists an element g in G such that $a = gbg^{-1}$. This is an equivalence relation (see proof given in class) called *conjugacy*. Hence there are equivalence classes of conjugate elements which we call *conjugacy classes*. In these notes we examine the conjucacy classes of S_n. The emphasis here is not on the proofs but on getting a "feeling" for what conjugacy means.

Consider the following elements of S_5. Can you tell which are in the same conjugacy classes? A= {(1,2), (3,4,5), (3,4), (1,2)(3,4) }
Without further information, it's NOT obvious. Now suppose we use the following theorem:

Two elements in S_n are conjugate iff they have the same cycle structure.

Memorize this now! It's very useful. E.g. Now we can immediately tell which two of the elements in set A (above) are conjugate. Which?

Is there an easy way to find an element g so that two elements are conjugate? YES! There is an easy way – it's stuff like this that makes theorems so useful. The proof of the above theorem is "constructive". That is we don't just prove such an element exists – we tell how to calculate an element g such that $a = gbg^{-1}$. This example illustrates the technique.

Consider a=(1,5)(3,4,2) and b=(3,1)(2,4,5). Now consider the following 2-row description of an element in S_5. Where'd I get this?

$$g = \begin{pmatrix} 1 & 5 & 3 & 4 & 2 \\ 3 & 1 & 2 & 4 & 5 \end{pmatrix}$$

1. Show that $a = gbg^{-1}$. Hint: First write g and g^{-1} in cycle notation.

2. Use the same idea to show that (1,2) and (3,4) are conjugate.

3. Are (3,4,5) and (1,2)(3,4) conjugate? Proof?

4. Are (3,4,5) and (1,2)(2,4) conjugate? Proof?