

Coset review and summary

Defn: If H is a subgroup of a group G ($H \leq G$) and $x \in G$ then the set xH is $\{xh|h \in H\}$. We call xH a left coset of H .

For this review to be useful, you need to be sure you know the answer to the “WHY’s” that appear below.

1. Suppose $aH = bH$. Then for some $h \in H$, $ah = b$ WHY?
2. If $ah = b$ then: $b^{-1}a$ and $a^{-1}b \in H$ WHY?
3. If aH and bH are two left cosets then either $aH = bH$ or $aH \cap bH = \emptyset$ WHY?
4. Suppose $H = bH$. Then $b \in H$ WHY?
5. There is a 1-1 onto map between any two left cosets of H . WHY? This means that finite left cosets are of the same size.
6. Every element of G is in some coset of H . WHY?
7. The number of left cosets of H in G is called the “index of H in G ” and is written: $[G : H]$.
8. $|H| = |aH|$. WHY?
9. For finite G we have: $|G| = [G : H] \cdot |H|$ WHY? And this tells us that the order of H divides the order of G .
10. We can also define right cosets and then find that there is a 1-1, onto map from the set of left cosets to the set of right cosets. We can also show that the order of a right coset is the same as the order of a left coset. WHY? WHY? WHY?