Homework on cardinality Due Feb 2

1. For each of the following pairs of sets give two proofs that the sets are equinumerous. One proof should be by Schroeder-Bernstein. The other proof should be by giving an explicit bijection.
   (a) S=[1,2] T=[5,8]
   (b) S=[1,2] T=[5,8)
   (c) S=[0,1) T=(0,1)
   (d) S=[0,1] T=(0,1)

2. For each of the following pairs of sets give a proof that the sets are equinumerous by giving an explicit bijection.
   (a) S=(0,1) T=$(0, \infty)$
   (b) S=(0,1) T=R

3. Prove the following:
   (a) If $S \subseteq T$ then $|S| \leq |T|$
   (b) $|S| \leq |S|$
   (c) If $|S| \leq |T|$ and $|T| \leq |U|$ then $|S| \leq |U|$
   (d) If S is finite then $|S| < \infty$

4. Let $\mathbb{A}$ denote all real numbers which are roots of polynomial equations with integer coefficients. The set $\mathbb{A}$ is known as: the algebraic numbers.
   (a) Prove $\mathbb{Q} \subseteq \mathbb{A}$
   (b) Follow the following outline to show that $\mathbb{A}$ is denumerable.
      i. First show that the set all polynomials of degree n is denumerable. (If this isn't obvious try doing n=1, n=2, and n=3 first.)
      ii. Then show the set of all polynomials (of any degree) with integer coefficients is denumerable.
      iii. Finally show that the set of all algebraic numbers is denumerable.
   (c) If the "transcendental numbers" are the non-algebraic reals, what can you say about the size of the set of transcendentals.